SCS 139

II.3 Induction and Inductance

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$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E}_L = -L\frac{di}{dt}$$

Office Hours:

Library (Rangsit)	Mon	16:20-16:50
BKD 3601-7	Wed	9:20-11:20

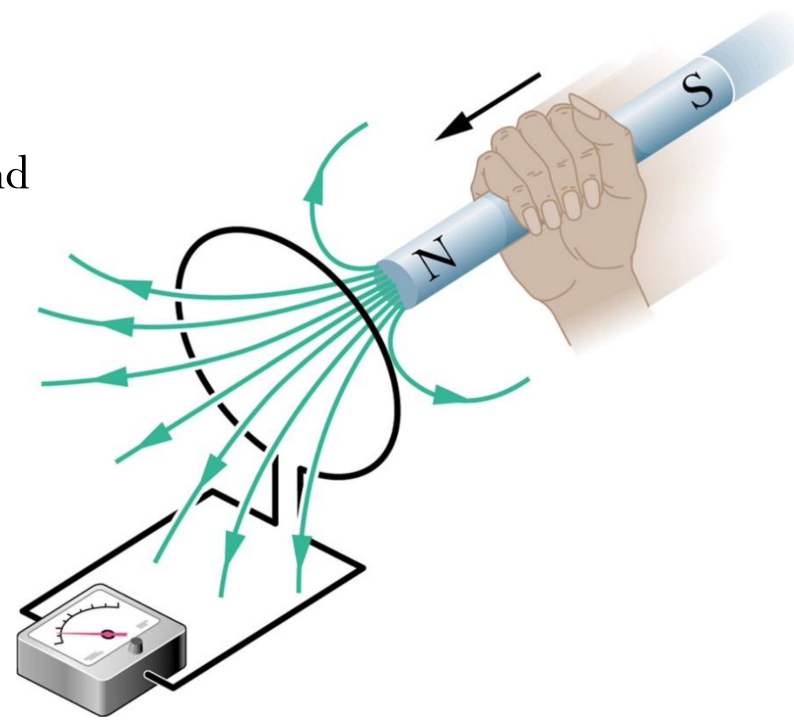
Review + New Fact

- Review
  - Force occurs when a charged particle moves through a magnetic field.
  - Force occurs when a current-carrying wire is placed in a magnetic field.
  - Magnetic field is found around a current-carrying wire.
- New Fact: Change in magnetic field can produce (**induce**) a current in a loop of wire

Experiment 1

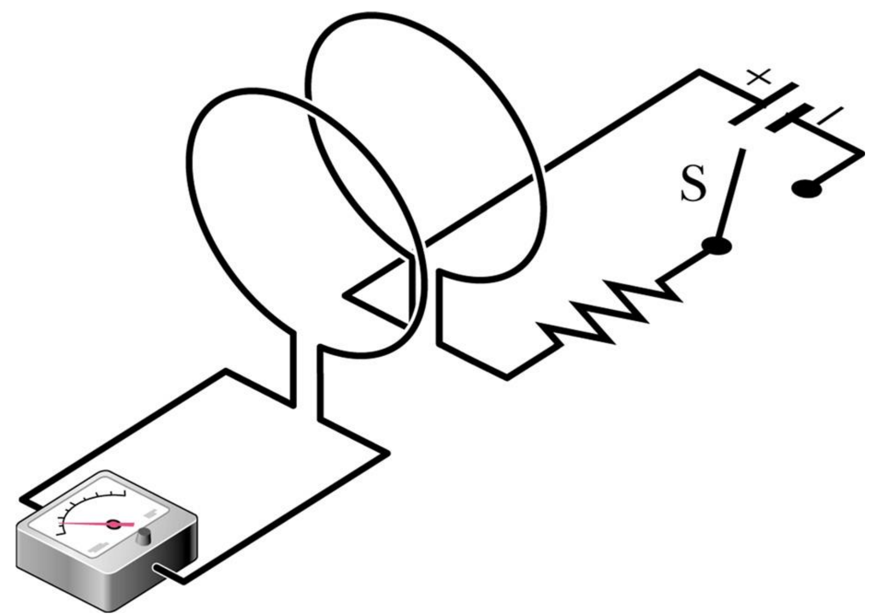
- Moving a magnet bar toward or away from a (conducting) loop of wire can produces (induces) a current in the loop.
  - The current produced in the loop is called an **induced current**.

- Observation:
  - Current only occurs when there is a **relative motion** between the loop and the magnet.
  - Faster motion produces a greater current
  - Direction (CW or CCW) of the (induced) current depends on the direction of motion and polarity of the magnet.



Experiment 2

- When the switch is suddenly closed (i.e. current flows through the right-hand loop) the ammeter will show a brief current appearing in the left-hand loop.
- When the switch is suddenly opened (i.e. no current flows through the right-hand loop) the ammeter will again show a brief current appearing in the left-hand loop, but in the opposite direction.



Induction

- The current produced in the loop is called an **induced current**.
- The work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf**.
- The process of producing the current and emf is called **induction**.
- **Faraday's law of induction**:
  An emf is induced in a loop when the
  (number of magnetic field line)
  amount of magnetic field that
  passes through the loop is changing.

Michael Faraday
(1791-1867)

Magnetic flux

- Need to quantify the amount of magnetic field that passes through the loop
- **Magnetic flux** through a loop enclosing an area *A*

Dot product

Vector of magnitude *dA* that is perpendicular to a differential area *dA*

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- Unit: weber (Wb)
  - 1 weber = 1 Wb = 1 $T \cdot m^2$

Changing magnetic flux through a coil

Here are the general means by which we can change the magnetic flux through a coil:

- Change the magnitude B of the magnetic field within the coil.
- Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).
- Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil (for example, by rotating the coil so that field $\vec{B}$ is first perpendicular to the plane of the coil and then is along that plane).

Faraday's law of induction

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

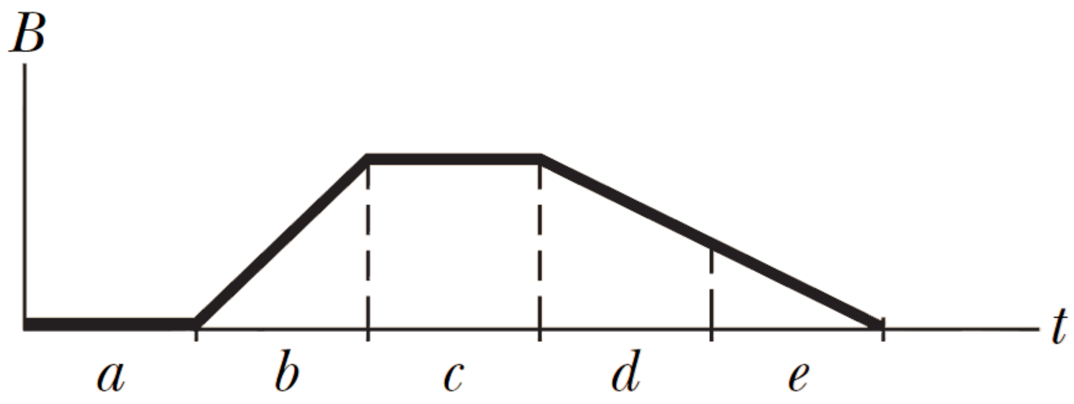
- The magnitude of the emf induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through the loop changes with time.
- The negative sign is there because the induced emf tends to oppose the flux change. (TBD)
- If we change the magnetic flux through a **coil of *N* turns**, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Assume that the coil is tightly wound (closely packed), so that the *same* magnetic flux passes through all the turns.

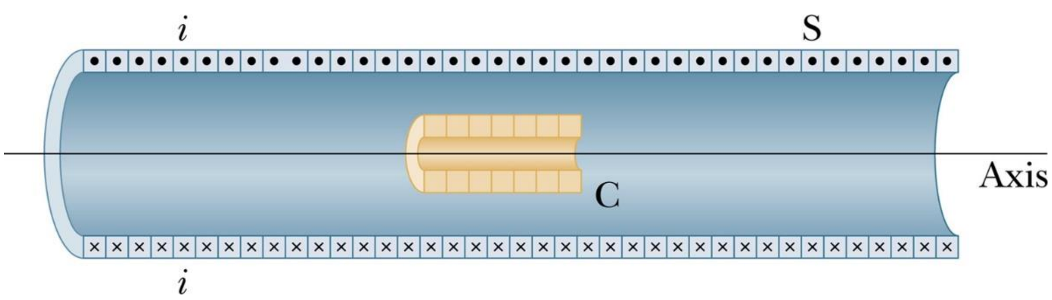
Exercise

- The graph gives the magnitude B(t) of a uniform magnetic field that exists throughout a conducting loop,with the direction of the field perpendicular to the plane of the loop.
- Rank the five regions of the graph according to the magnitude of the emf induced in the loop, greatest first.



Ex: Induced emf due to a solenoid

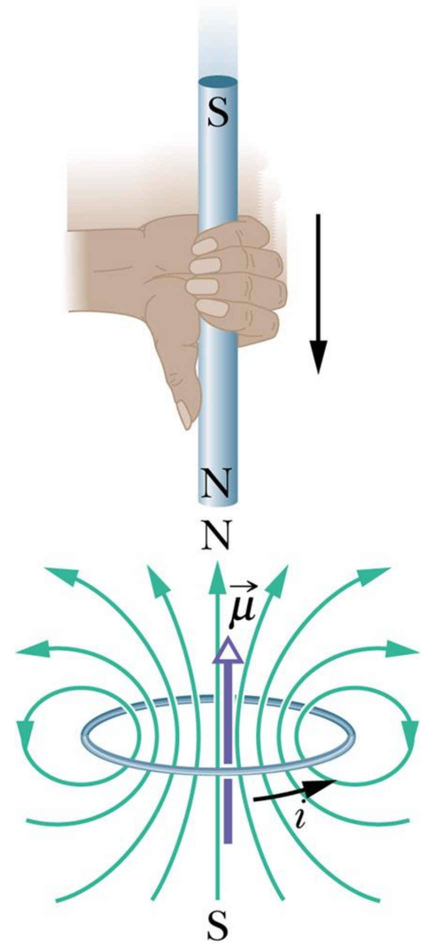
- A long solenoid has 220 turns/cm and carries a current i =1.5 A; its diameter D is 3.2 cm.
- At the center we place a 130 turn closely packed coil C of diameter d = 2.1 cm. The current in the long solenoid is reduced to zero at a constant rate in 25 ms.
- What is the magnitude of the induced emf in coil C?



Lenz's law

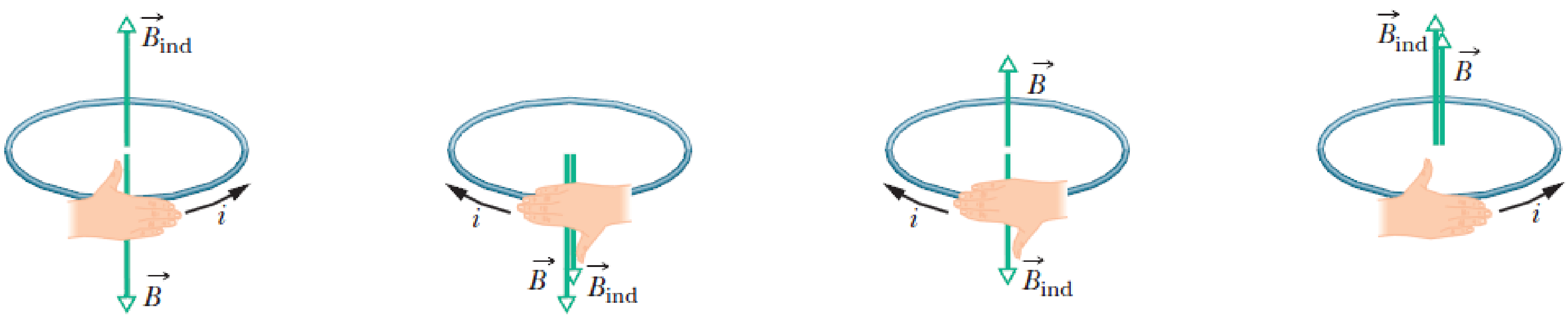
- "An induced current has a direction such that the magnetic field due to the current **opposes** the change in the magnetic flux that induces the current."

Heinrich Lenz
(1804-1865)



Ex: Lenz's law

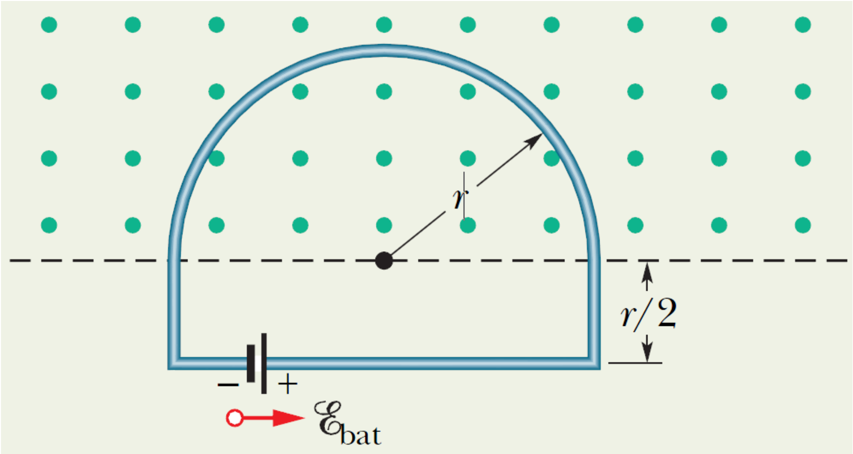
Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*	Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*	Increasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*	Decreasing the external field $\vec{B}$ induces a current with a field $\vec{B}_{ind}$ that *opposes the change.*



Note carefully that the flux of $\vec{B}_{ind}$ always opposes the *change* in the flux of $\vec{B}$.
Does *not* mean that $\vec{B}_{ind}$ always points opposite $\vec{B}$.

Ex. Induction due to a changing B

- Consider a conducting loop consisting of a half-circle of radius $r = 0.20$ m and three straight sections. The half-circle lies in a uniform magnetic field that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds.
- An ideal battery with emf $\mathcal{E}_{bat} = 2.0$ V is connected to the loop.
- The resistance of the loop is 2.0 Ω

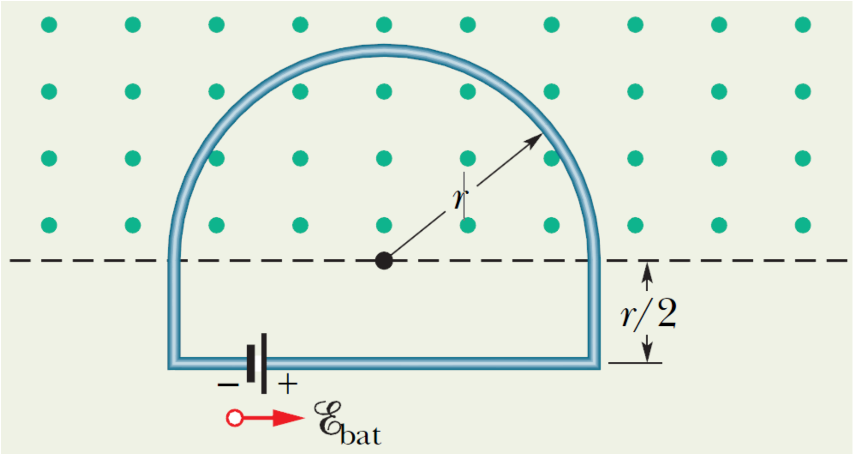


Ex. Induction due to a changing *B*

- $r = 0.20$ m, $B = 4.0t^2 + 2.0t + 3.0$, $\mathcal{E}_{bat} = 2.0$ V, $R = 2.0\ \Omega$

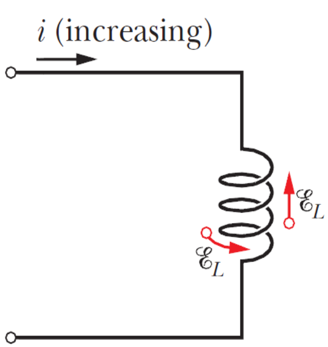
a) What are the magnitude and direction of the emf $\mathcal{E}_{ind}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

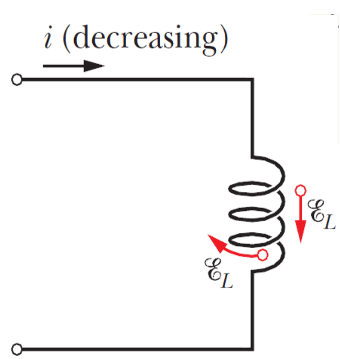
b) What is the current in the loop at $t = 10$ s?



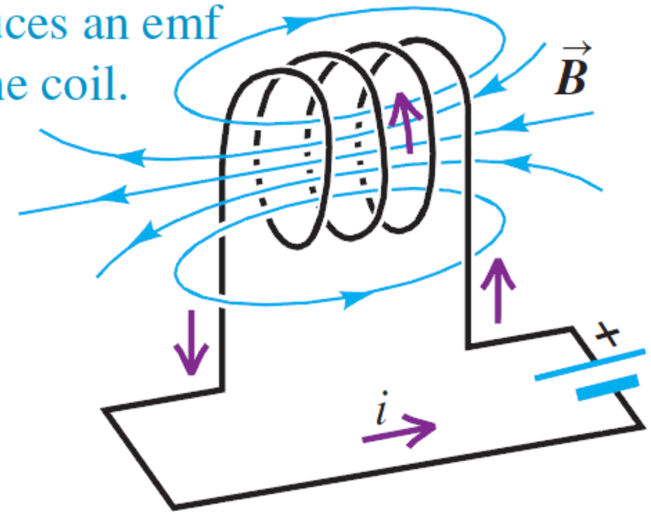
Self-Induction

- An induced emf appears in any coil in which the current is changing.
- This process is called **self-induction**.
- The emf that appears is called a **self-induced emf**.
- Still obeys Faraday's law and Lenz's law.





Self-inductance: If the current i in the coil is changing, the changing flux through the coil induces an emf in the coil.



Inductors and Inductance

- An **inductor** is an electrical component typically made by coiling a conductor around a core.
  - **Solenoid** is our basic type of inductor.
- The **inductance** of the inductor is

the number of turns

$$L \equiv \frac{N\Phi_B}{i}$$

- Unit: henry
  - 1 henry = 1 H = 1 T·m^2/A.

Joseph Henry (1797-1878)

$$L_{\text{solenoid}} = \frac{\overset{N}{(n\ell)}\overbrace{(\mu_0 i n)}^{B}A}{i} = n^2\mu_0\ell A$$

Inductor: self-induced emf

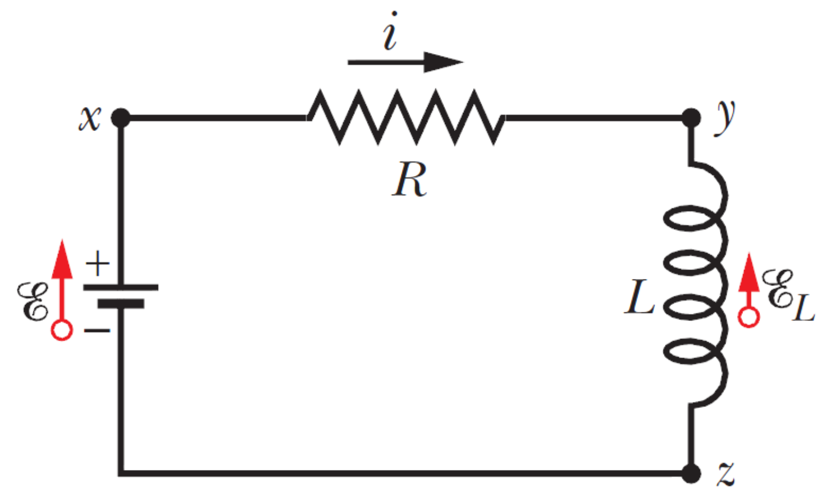
- Let's combine self induction and inductance.
- In any inductor (such as a solenoid) a self-induced emf appears whenever the current changes with time.

$$\mathcal{E}_L \underset{\text{Faraday's law}}{=} -N\frac{d\Phi_B}{dt} = -\frac{d\left(N\Phi_B\right)}{dt} \underset{\text{Inductance } L = \frac{N\Phi_B}{i}}{=} -\frac{d\left(Li\right)}{dt} = -L\frac{di}{dt}$$

- The direction can be obtained by Lenz's law.
- For an ideal inductor with negligible resistance, the magnitude of the potential difference V_{L} across the inductor is equal to the magnitude of the self-induced emf $\mathcal{E}_L$.

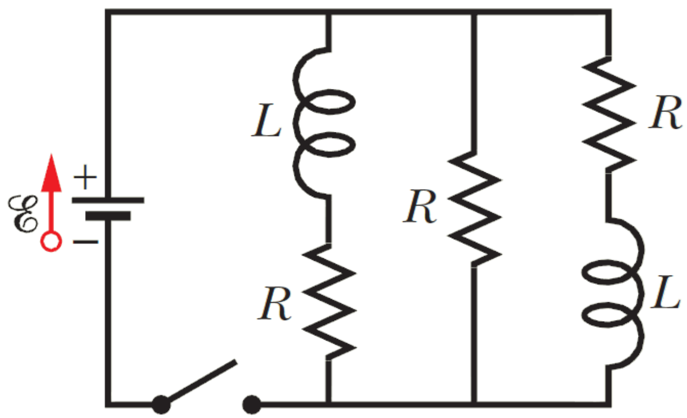
RL Circuit

- Kirchhoff's voltage law



Example. *RL* circuit

- Consider a circuit that contains three identical resistors with resistance $R = 9.0$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathcal{E} = 18$ V.



- What is the current i through the battery just after the switch is closed?

Example. *RL* circuit

- What is the current *i* through the battery long after the switch has been closed?